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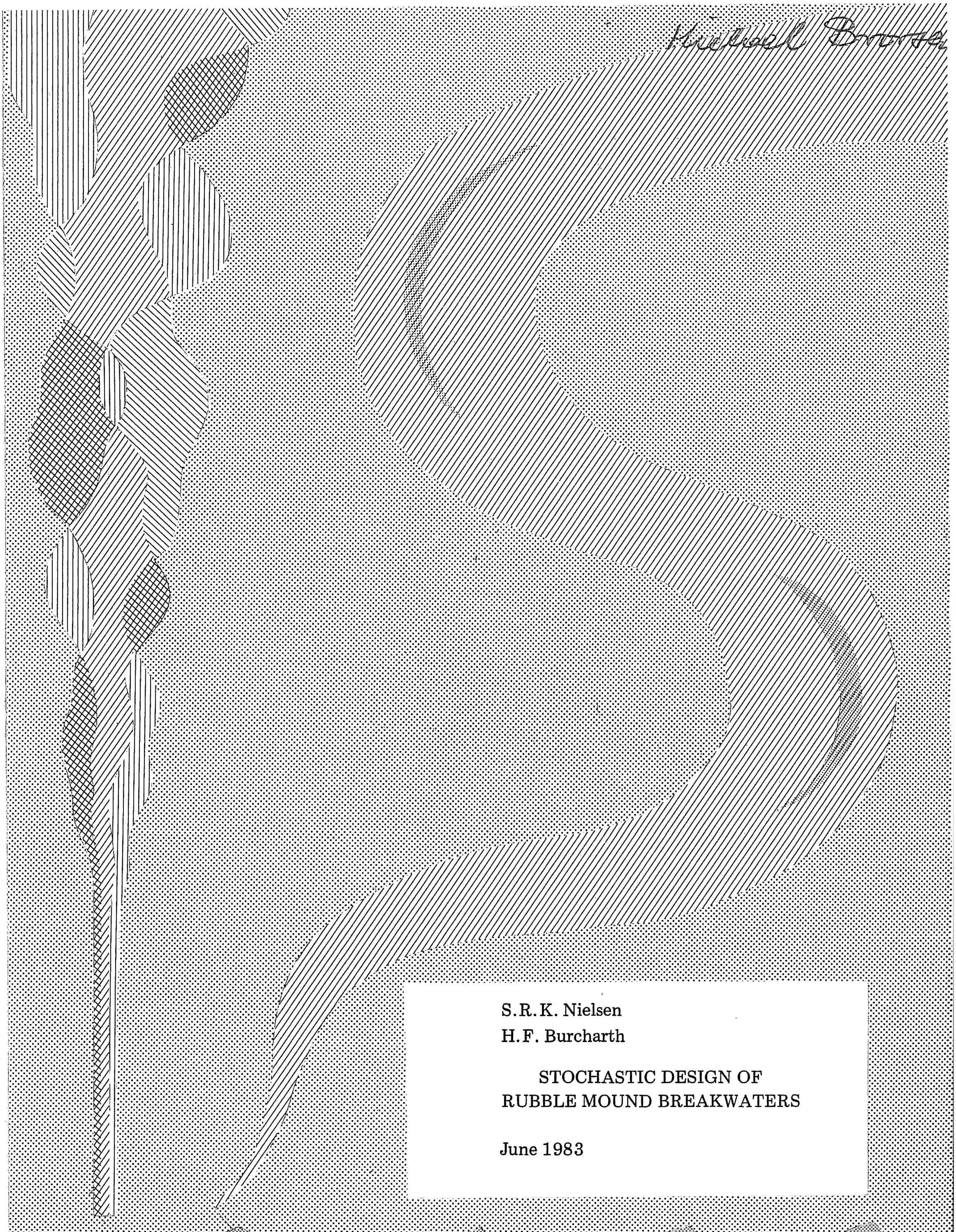

S.R.K. Nielsen
H.F. Burcharth

STOCHASTIC DESIGN OF RUBBLE MOUND BREAKWATERS

June 1983

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By

S. R. K. Nielsen [1] and H. F. Burcharth [2]

ABSTRACT

The paper presents a level III reliability method from which the armour layer of rubble mound breakwaters can be designed, so that the total costs of construction price and expected maintenance expenses are minimized. Since the physics of the wave-structure interaction are not yet fully understood the paper puts emphasis on the application of probability theory rather than on hydraulic aspects.

The armour layer is considered a structural system made up of a number of basic unit areas. For each basic area a causal relationship must be established by modeltests between the wave loadings and their effects in terms of damage rates. Variability of test results due to uncontrolled parameters is assessed by a variance analysis of the damage rates. Failure is, in the mathematical context, taken as the failure of at least one basic unit, i.e. the system is of the weakest-link-type.

Two failure modes of a basic unit area are considered. The armour blocks may be destroyed by rocking, either due to structural failure from heavy rocking or accumulated fatigue damages caused also by smaller storms or to failure of the unit area because armour units are displaced, so that the core of the breakwater is uncovered. The distribution function of lifetimes of each basic unit area can then be calculated, assuming that the number of storms capable of giving rise to any damage to the armour layer in any failure mode are specified by homogeneous Poisson counting processes.

From the reliability analysis the expected repair price for future failures can be calculated. It is demonstrated that the total price, made up of construction price and future maintenance expenses, has a minimum at a certain design.

The outlined theory is finally demonstrated by a numerical example.

1. INTRODUCTION

A rubble mound breakwater can fail due to instability of the armour layer, damage to the wave screen, erosion of the front toe (on minor breakwaters), damage to the back slope to overtopping etc. Some of these failure modes are correlated, e.g. the stability of the armour layer is reduced if the front toe is damaged. However, the interaction of these modes are not yet well understood, and hence a total reliability analysis of a breakwater with due consideration to all failure modes requires extensive model testing to establish all relevant relationships. An analysis can then, of course, be performed based on estimated distribution functions. The present paper describes a level III reliability analysis of the unimodal failure of the armour layer. The method takes into account the minimization of the construction price plus the expected repair expenses from future failures within the stipulated lifetime of the structure. The unimodal reliability analysis of other failure modes can be treated in a similar way.

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Both hydraulic and mechanical stability of the armour layer should be considered. The most important parameters affecting these stability requirements have been listed below.

The hydraulic stability is influenced by

I Armour unit geometry and relative density.
Armour layer thickness and packing density.
Filler layer permeability and thickness.
Core permeability.
Cross section profile (slope angle, berms, wave walls, etc.).

II Deep and shallow water climate.
Joint probability distributions of wave height, -period, and -direction, wave grouping, long and short term statistics and persistance of waves, shoaling effects.

III Bottom bathymetry.

IV Water level variations (storm surge, tides, etc.).

As seen the number of parameters are large and their separate impact on the stability is not always well established. Hence a general set of formulae determining the hydraulic stability does not exist and it may take years before the necessary research and datacollection have been finished. The lack of knowledge has led to a semiempirical approach, where a number of important parameters are combined into some overall parameters, identified by physical reasonings and dimensional analysis. The response of the structure in terms of damage rates, overtopping, etc., and rocking percentages is then studied in model tests with the rest of the parameters at reference values. Results from armour stability tests can be as shown on figure 10.

The hydraulic stability will gradually be reduced during time due to abrasion from suspended materials. This can be modelled by a function which increases the damage rates with time. This function is determined from hydraulic stability tests with units with different degrees of rounding and from site experience. Weathering and thermal and chemical reactions also cause rounding which can be modelled similar to abrasion and which will gradually reduce the strength of the armour units. Further repeated loadings causing stress ranges above a certain minimum level will cause fatigue and hence further reduce the strength. The mechanical stability of the armour units is ensured when the residual strength exceeds the stresses induced by the waveloading during the intended lifetime of the structure.

The two failure modes of the armour layer dealt with in the present model correspond to the lack of mechanical and hydraulic stability. The damage rates in both failure modes are determined by model tests. The scatter of these results due to uncontrolled parameters is considered in the final variance analysis of the damage rates.

2. BASIC ASSUMPTIONS AND RESULTS

In most cases model tests for each typical breakwater-seabed profile are performed in a wave flume on a limited section of a breakwater. The local weakenings of the armour layer in similar sections are not considered and hence the reliability of the breakwater is over- or underestimated if the tests are not repeated several times. In order to deal with this scale effect rationally, the armour layer is considered as a system of M basic unit areas, each with separate strength and loading. Damage rates to specific waveloadings are determined for some or all of these basic areas. It is taken as failure of the breakwater armour layer when at least one basic unit area fails, i.e. the system can be identified as a weakest-link-system.

Two failure modes of a basic unit area are considered. The first failure mode, $k = 1$ of the armour layer specifies the fracture of the armour units from rocking either due to structural failure from heavy rocking or accumulated fatigue damages from smaller storms. This failure mode is relevant especially for the slender complex types of units such as Dolosse and Tetrapods. The second failure mode, $k = 2$ indicates displacement of armour units at least of the magnitude of one characteristic diameter, resulting in exposure of the underlayers to the waves.

The accumulated damage percentage processes $\{D_{1,i}(t)\}$ and $\{D_{2,i}(t)\}$ during the interval $[0, t]$ signifies the relative number of armour blocks in the i'th basic unit area, failing in mode 1 and 2, respectively.

These quantities are assumed on the form

$$D_{1,i}(t) = \sum_{n=1}^{N_{1,i}(t)} \Delta D_{1,i,n} \cdot s_{1,i}(\tau_{1,i,n}) \tag{1}$$

$$D_{2,i}(t) = \sum_{n=1}^{N_{2,i}(t)} \Delta D_{2,i,n} \cdot s_{2,i}(\tau_{2,i,n}) \tag{2}$$

Damages at basic unit area i in failure mode k take place at random times $0 \leqslant \tau_{k,i,1} < \ldots\ldots\ldots < \tau_{k,i,N_{k,i}(t)} \leqslant t$. $\{N_{1,i}(t)\}$ and $\{N_{2,i}(t)\}$ are homogeneous Poisson counting processes specifying the random number of such damage increments in the interval $[0,t]$, ref. [4]. The intensities $\lambda_{k,i}$ of the counting processes indicate the expected number of storms per unit of time capable of giving rise to any damage contributions in mode k at basic unit area i.

The damage increments $\Delta D_{k,i,n}$, $n \in 1, 2, \ldots$, are stochastic variables, assumed to be mutually independent and identically distributed as the stochastic variable $\Delta D_{k,i}$. These quantities will be further explained in a succeding section. The samples of $\Delta D_{k,i}$ are positive. Hence the realizations of (1) and (2) are non-decreasing functions with probability 1.

$s_{k,i} : [0,t] \curvearrowright R_+$, $k \in 1, 2$ are deterministic, monotonic, increasing functions of time, specifying long term tendencies of increased damage rates, e.g. due to abrasion. These functions are normalized as follows

$$s_{k,i}(0) = 1 \tag{3}$$

A typical realization of the accumulated damage percentage processes is shown on figure 1.

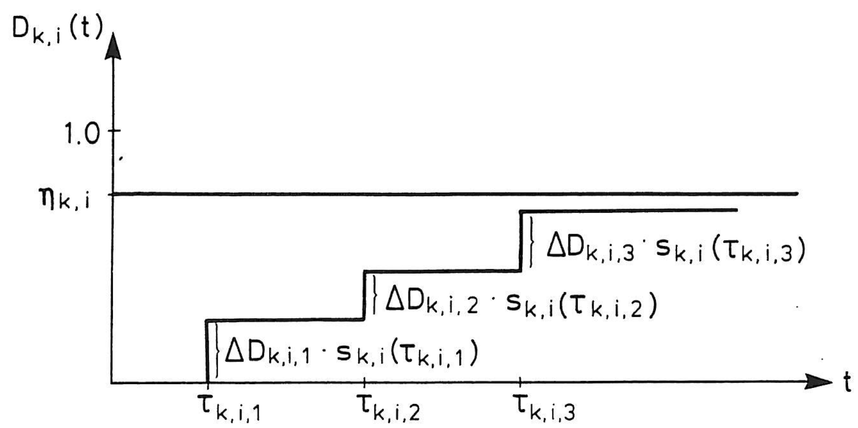


Figure 1: Realization of accumulated damage percentage process.

The distribution function of the accumulated damage percentage $D_{k,i}(t)$ at time t can now be determined. Actually this quantity has the characteristic function

$$M_{D_{k,i}(t)}(\theta) = \exp\left(\lambda_{k,i} \int_0^t (M_{\Delta D_{k,i}}(s_{k,i}(\tau)\,\theta) - 1)\, d\tau\right) \tag{4}$$

where $M_{\Delta D_{k,i}} : R \cap C$ is the characteristic function of $\Delta D_{k,i}$.

The expectation and variance of $D_{k,i}(t)$ then become

$$\mu_{D_{k,i}}(t) = E[\Delta D_{k,i}] \cdot \lambda_{k,i} \cdot \int_0^t s_{k,i}(\tau)\, d\tau \tag{5}$$

$$\sigma^2_{D_{k,i}}(t) = E[\Delta D^2_{k,i}] \cdot \lambda_{k,i} \cdot \int_0^t s^2_{k,i}(\tau)\, d\tau \tag{6}$$

The distribution of $D_{k,i}(t)$ is of the mixed type with a concentrated, although diminishing probability mass $\epsilon_{k,i}(t)$ at $D_{k,i}(t) = 0$:

$$\epsilon_{k,i}(t) = P(D_{k,i}(t) = 0) = P(N_{k,i}(t) = 0) = \exp(-\lambda_{k,i} \cdot t) \tag{7}$$

The remaining probability mass $(1 - \epsilon_{k,i}(t))$ is continuously distributed as specified by the frequency function $f^{\circ}_{D_{k,i}(t)}(x)$, with expectation $\mu^{\circ}_{D_{k,i}}(t)$ and standard deviation $\sigma^{\circ}_{D_{k,i}}(t)$, see figure 2

$$\mu^{\circ}_{D_{k,i}}(t) = \frac{\mu_{D_{k,i}}(t)}{1 - \epsilon_{k,i}(t)} \tag{8}$$

$$\sigma^{\circ}_{D_{k,i}}(t) = \left(\frac{\sigma^2_{D_{k,i}}(t)}{1 - \epsilon_{k,i}(t)} - \frac{\epsilon_{k,i}(t)}{(1 - \epsilon_{k,i}(t))^2}\, \mu^2_{D_{k,i}}(t)\right)^{\frac{1}{2}} \tag{9}$$

The moments (8) and (9) provide that $D_{k,i}(t)$ will have the moments (5) and (6).

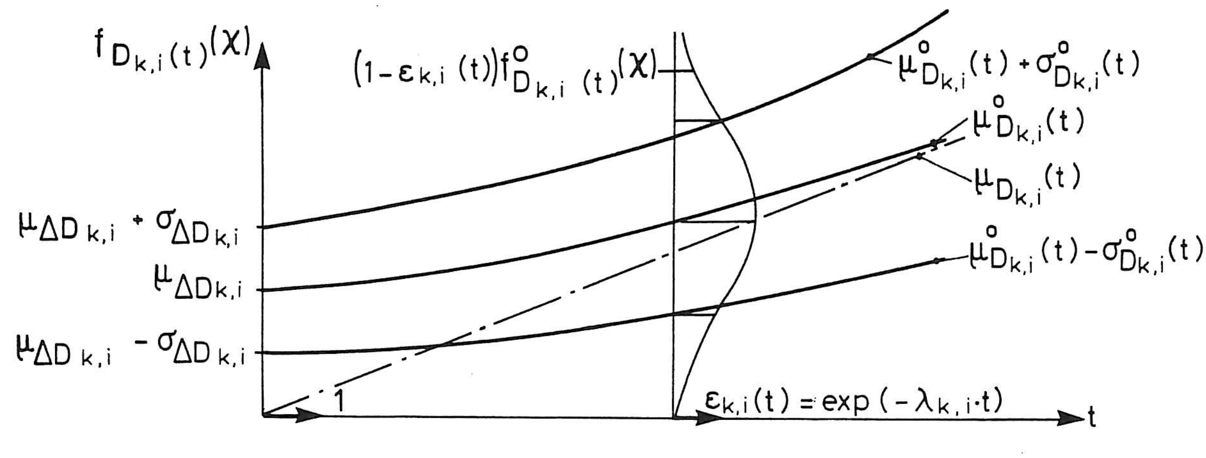


Figure 2: First order probability density function of accumulated damage percentage process.

The distribution function of $D_{k,i}(t)$ approaches a normal distribution, as $t \to \infty$, due to the central limit theorem. Instead of the exact result (4), it then seems reasonable to apply the following approximation, which will be asymptotically correct

$$F_{D_{k,i}(t)}(\chi) \cong (1 - \epsilon_{k,i}(t))\ \Phi\left(\frac{\chi - \mu^{\circ}_{D_{k,i}}(t)}{\sigma^{\circ}_{D_{k,i}}(t)}\right) + \epsilon_{k,i}(t) \tag{10}$$

Φ indicates the distribution function of a standardized normal variable.

(4) requires the exact distribution function of $\Delta D_{k,i}$.

The approximation (10) only requires that the first and second order moments of $\Delta D_{k,i}$ are known, cf. (5) and (6).

Failure of basic unit area i occurs, when the accumulated damage percentages $\{D_{1,i}(t)\}$ and $\{D_{2,i}(t)\}$ for the first time exceed the allowable limits $\eta_{1,i}$ and $\eta_{2,i}$, which can be specified separately for any basic area, depending on its position and importance. The reliability problem of a certain basic unit area then becomes an ordinary first passage probability problem.

The stochastic variables $D_{1,i}(t)$ and $D_{2,i}(t)$ can be considered mutually independent at large time intervals $[0,t]$. Because the realizations of the processes $\{D_{k,i}(t)\}$, $k \in 1,2$, are non-decreasing with probability 1, it follows that the distribution function of the first passage time $L_{1,i}$, i.e. the elapsed time from the time of construction until the first failure of the basic unit area i occurs, can be calculated as follows

$$P(L_{1,i} > t) = P(D_{1,i}(t) \leqslant \eta_{1,i} \wedge D_{2,i}(t) \leqslant \eta_{2,i}) \Rightarrow$$

$$F_{L_{1,i}}(t) = 1 - F_{D_{1,i(t)}}(\eta_{1,i})\ F_{D_{2,i(t)}}(\eta_{2,i}) \tag{11}$$

(11) follows from the fact that the event $\{L_{1,i} > t\}$ occurs if and only if no failure takes place in any of the two failure modes during the interval $[0,t]$. The frequency function of $L_{1,i}$ is determined by differentiation of (11) with respect to t. The result becomes

$$f_{L_{1,i}}(t) = -g_{1,i}(t) \cdot F_{D_{2,i}(t)}(\eta_{2,i}) - g_{2,i}(t)\ F_{D_{1,i}(t)}(\eta_{1,i}) \tag{12}$$

$$g_{k,i}(t) = \frac{\partial}{\partial t} F_{D_{k,i}(t)}(\eta_{k,i}) = -\lambda_{k,i}\,\epsilon_{k,i}(t)\left(1 - \Phi\left(\frac{\eta_{k,i} - \mu^{\circ}_{k,i}(t)}{\sigma^{\circ}_{k,i}(t)}\right)\right)$$

$$- (1 - \epsilon_{k,i}(t))\ \varphi\left(\frac{\eta_{k,i} - \mu^{\circ}_{k,i}(t)}{\sigma^{\circ}_{k,i}(t)}\right) \frac{\dot{\mu}^{\circ}_{k,i}(t)\,\sigma^{\circ}_{k,i}(t) + \dot{\sigma}^{\circ}_{k,i}(t)\,(\eta_{k,i} - \mu^{\circ}_{k,i}(t))}{(\sigma^{\circ}_{k,i}(t))^2} \tag{13}$$

$\varphi(\cdot)$ signifies the frequency function of a standardized normal variable, and $\dot{\mu}^{\circ}_{k,i}$ and $\dot{\sigma}^{\circ}_{k,i}$ are the time derivatives of the functions (8) and (9).

For $t = 0$ we have

$$f_{L_{1,i}}(0) = \sum_{k=1}^{2} \lambda_{k,i} \cdot \left(1 - \Phi\left(\frac{\eta_{k,i} - \mu_{\Delta D_{k,i}}}{\sigma_{\Delta D_{k,i}}}\right)\right) \tag{14}$$

where $\mu_{\Delta D_{k,i}}$ and $\sigma_{\Delta D_{k,i}}$ are the expected value and standard deviation of $\Delta D_{k,i}$.

On figure 3 some first passage density curves originating from (12) are shown. There may or may not be a local maximum depending primarily on the magnitude of the variational coefficient $V[\Delta D_{k,i}] = \sigma_{\Delta D_{k,i}} / \mu_{\Delta D_{k,i}}$ of incremental damages. When $V[\Delta D_{k,i}]$ is large, no local maximum occurs, see figure 3 b.

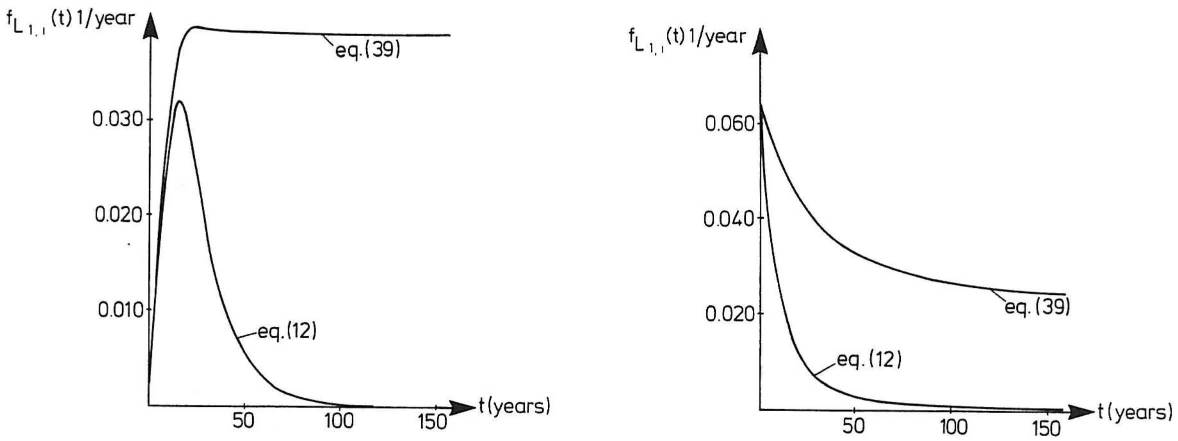


Figure 3: *The first passage probability densities and the expected damage rate in the basic unit area i, as calculated from equation (12) and (39). $\lambda_{k,i} = 0.1$, $\eta_{k,i} = 0.1$, $s_{k,i}(t) \equiv 1.0$. a) $V[\Delta D_{k,i}] = 1.0$. b) $V[\Delta D_{k,i}] = 5.0$.*

The distribution function of the first passage time $L_{1,A}$ of the entire armour layer can now be calculated, assuming the lifetimes of all basic units to be mutually independent stochastic variables. This assumption will be valid if the elapsed time interval as well as the magnitude of the selected basic unit areas is sufficiently large. Hence

$$F_{L_{1,A}}(t) = 1 - \prod_{i=1}^{M} (1 - F_{L_{1,i}}(t)) \qquad (15)$$

3. MODELLING OF DAMAGE INCREMENT

The wave condition in the basic unit area i is specified by the parameters $(H_{s,i}, T_{p,i})$, where $H_{s,i}$ is the significant wave height, and $T_{p,i}$ is the peak period (most probable wave period). A storm is characterized by the growth and the succeeding decrease of wave heights, cf. figure 4. The maximum wave height $H_{s,i,max}$ is assumed to be Weibull distributed, i.e.

$$F_{H_{s,i,max}}(h) = 1 - \exp(-(\frac{h - h_1}{h_2})^{h_3}), \quad h \in [h_1, \infty[\qquad (16)$$

The significant wave heights within a certain storm, on condition of the maximum wave height $H_{s,i,max} = \chi$, is assumed to be uniformly distributed in the interval $[h_1, \chi]$, i.e.

$$F_{H_{s,i} \mid H_{s,i,max}}(h \mid \chi) = \begin{cases} 0 & , \quad h \in [\,0, h_1\,] \\ \dfrac{h - h_1}{\chi - h_1} & , \quad h \in \,]\,h_1, \chi\,] \\ 1 & , \quad h \in \,]\,\chi, \infty\,[\end{cases} \tag{17}$$

The uniform distribution is tantamount to the triangular growth and decrease curve shown on figure 4.

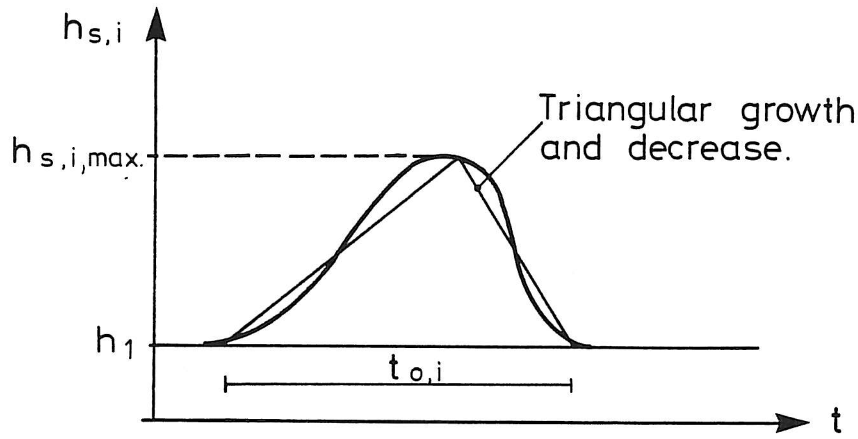


Figure 4: Variation of significant wave height during a single storm.

Rocking induces impact stresses in the armour units. In the present paper this is considered the main reason of fatigue damage, and possible damage contributions from wave loadings incapable of rocking the armour units are ignored. Further it is as a first approximation assumed that any armour unit rocking in a storm also will be rocking in a later and severer storm. At a certain limiting storm level $\eta^{\circ}_{1,i}$ percentage of the armour units will be rocking. Obviously, the accumulated fatigue damage will be smallest in the last activated armour unit. Hence, when this unit is failing due to accumulated fatigue damage exactly $\eta^{\circ}_{1,i}$ percentage of the armour units must have failed.

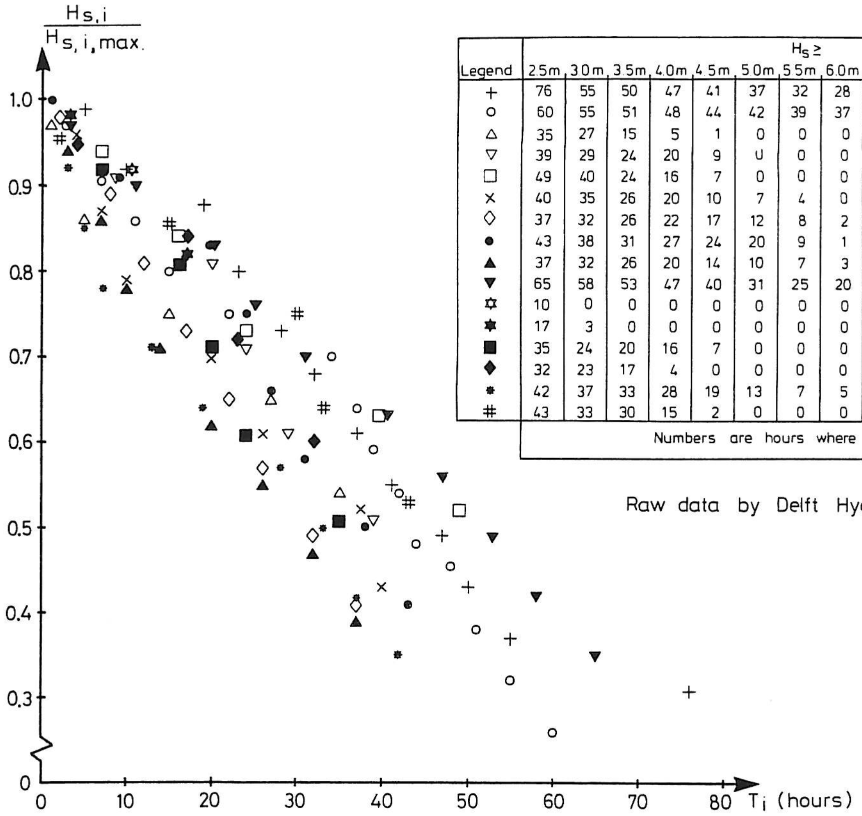
$\Delta D_{1,i}$ signifies the fatigue damage increment in the last activated armour unit among all $\eta^{\circ}_{1,i}$ percentage of armour units, rocking at the limiting storm level. For a storm with $H_{s,i,max} = \chi$, this quantity is assumed on the form

$$\Delta D_{1,i} = \int_{h_1}^{\chi} \frac{dt}{T_{0,i}(\Delta\bar{\sigma})} \tag{18}$$

dt is the time interval with significant wave heights in the interval $]h, h + dh]$. In accordance with the assumption (17), dh and dt will be linearly dependent

$$dt = t_{0,i} \frac{dh}{\chi - h_1} \tag{19}$$

$t_{0,i}$ is the duration of a storm with $H_{s,i} = h_1$, see figure 4. (17) and (19) have been based on a great number of data samples, cf. figure 5. As seen $t_{0,i}$ turns out to be independent on χ. The data represent the largest storms in a 20 years period for a certain location. The smaller storms represent sea states where movements are negligible in respect of fatigue damage.



Legend	$H_s \geq$ 2.5m	3.0m	3.5m	4.0m	4.5m	5.0m	5.5m	6.0m	6.5m	7.0m	7.5m	8.0m	8.5m	9.0m	max H_s m
+	76	55	50	47	41	37	32	28	23	19	10	5	0	0	8.11
○	60	55	51	48	44	42	39	37	34	22	15	11	7	3	9.32
△	35	27	15	5	1	0	0	0	0	0	0	0	0	0	4.64
▽	39	29	24	20	9	U	0	0	0	0	0	0	0	0	4.92
□	49	40	24	16	7	0	0	0	0	0	0	0	0	0	4.78
×	40	35	26	20	10	7	4	0	0	0	0	0	0	0	5.72
◇	37	32	26	22	17	12	8	2	0	0	0	0	0	0	6.15
●	43	38	31	27	24	20	9	1	0	0	0	0	0	0	6.03
▲	37	32	26	20	14	10	7	3	0	0	0	0	0	0	6.37
▼	65	58	53	47	40	31	25	20	11	3	0	0	0	0	7.19
✡	10	0	0	0	0	0	0	0	0	0	0	0	0	0	2.73
✸	17	3	0	0	0	0	0	0	0	0	0	0	0	0	3.06
■	35	24	20	16	7	0	0	0	0	0	0	0	0	0	4.90
◆	32	23	17	4	0	0	0	0	0	0	0	0	0	0	4.19
✹	42	37	33	28	19	13	7	5	3	0	0	0	0	0	7.06
#	43	33	30	15	2	0	0	0	0	0	0	0	0	0	4.67

Numbers are hours where H_s is exceeded

Raw data by Delft Hydraulics Laboratory.

Figure 5: Duration of storms T_i, with significant wave height exceeding level $H_{s,i}$.

Basically it is assumed that the stress ranges from a single wave within a certain armour unit can be identified by only one parameter. The average value of this quantity at a time where the storm is specified by $(H_{s,i}, T_{p,i})$ is termed $\Delta\bar{\sigma}_i$. The dependency of $\Delta\bar{\sigma}_i$ on $(H_{s,i}, T_{p,i})$ can generally only be established by model- or full scale tests. The following explicit relationship has been assumed in what follows

$$\Delta\bar{\sigma}_i = \Delta\bar{\sigma}_{i,1} \frac{H_{s,i} - h_{s,i,0}}{h_{s,i,1} - h_{s,i,0}} \quad , \quad H_{s,i} \in [h_{s,i,0} \, , \, \infty[\tag{20}$$

$\Delta\bar{\sigma}_{i,1}$ is the average stress range at the wave height $h_{s,i,1}$.

$\Delta\bar{\sigma}_{i,1}$ depends to some extent on the magnitude of the armour units. $h_{s,i,0}$ is the wave height, at which $\eta^{\circ}_{1,i}$ percentage of the armour units is rocking.

$T_{0,i}(\Delta\bar{\sigma}_i)$ is the fatigue life of the armour units in a wave condition where the average stress range is $\Delta\bar{\sigma}_i$. Consequently (18) in combination with (1) is a formulation of the conventional Palmgren-Miner accumulated damage theory.

$T_{0,i}$ as a function of $\Delta\bar{\sigma}_i$ is a material property, which has been assumed on the form

$$T_{0,i}(\Delta\bar{\sigma}_i) = T_{z,i} \cdot \begin{cases} \dfrac{1}{\left(\dfrac{\Delta\bar{\sigma}_i}{\bar{\sigma}_T}\right)^{m_i}} & , \quad \Delta\sigma_{i,min} \leq \Delta\bar{\sigma}_i < \infty \\ \infty & , \quad 0 \leq \Delta\bar{\sigma}_i < \Delta\sigma_{i,min} \end{cases} \tag{21}$$

$T_{z,i}$ is the expected zero crossing period, $\bar{\sigma}_T$ is the expected dynamic tensile strength, $\Delta\sigma_{i,min}$ is the minimum stress range capable of inducing any fatigue damage in the material, and m_i is a material parameter. $\Delta\sigma_{i,min}$ is obtained at a certain limiting wave height $h_{i,min}$. As seen from (20), $h_{i,min} > h_{s,i,0}$.

In double logaritmic mapping (21) is identified as an ordinary Wöhler curve, cf. figure 6.

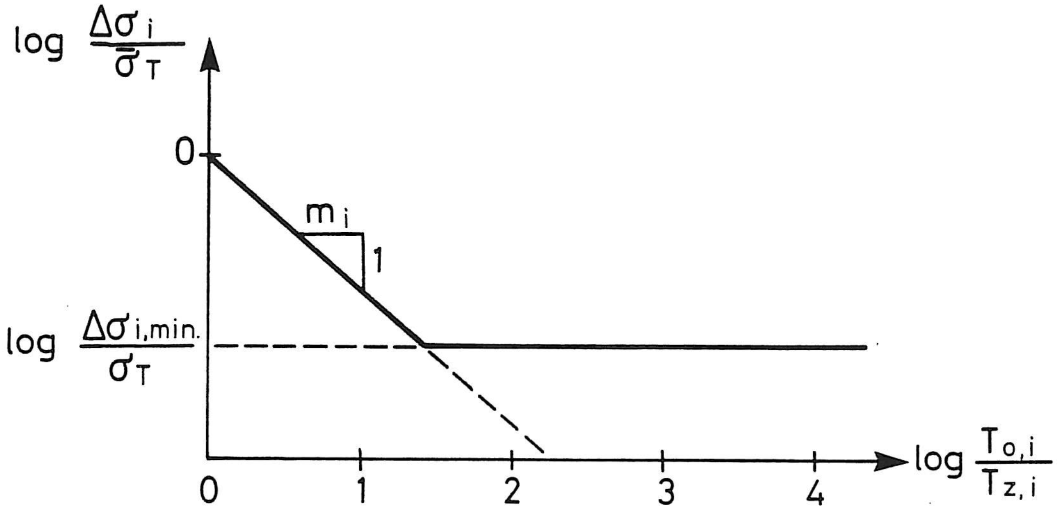


Figure 6: *Wöhler curve for armour units.*

$T_{p,i}$ is assumed to be fully dependent on the significant wave height $H_{s,i}$ as given by the following explicit relationship

$$T_{p,i} = t_{p,i,1}\left(\frac{H_{s,i}}{h_{s,i,1}}\right)^{h_4} \tag{22}$$

$t_{p,i,1}$ is the wave period at the referential wave height $h_{s,i,1}$.

The zero crossing period $T_{z,i}$ is related to the peak period and significant wave height $H_{s,i}$ as follows

$$T_{z,i} = t_{z,i,1}\,\frac{T_{p,i}}{t_{p,i,1}} = t_{z,i,1}\left(\frac{H_{s,i}}{h_{s,i,1}}\right)^{h_4} \tag{23}$$

On condition of the sample $H_{s,i,max} = \chi$, the displacement damage increment $\Delta D_{2,i}$ is suggested in the form

$$\Delta D_{2,i} = \int_{h_1}^{\chi} \frac{dt}{t_{2,i}} \cdot \Delta D^{\circ}_{2,i}\,(h, t(h))$$

$$= \frac{t_{0,i}}{t_{2,i}} \cdot \frac{1}{\chi - h_1} \int_{h_1}^{\chi} \Delta D^{\circ}_{2,i}\,(h, t(h))\,dh \tag{24}$$

where (19) has been applied.

$\Delta D^{o}_{2,i}$ ($H_{s,i}$, $T_{p,i}$) is the relative number of blocks within the basic unit area i, displaced at least one characteristic diameter relative to their initial position in a storm ($H_{s,i}$, $T_{p,i}$) during the interval $t_{2,i}$. Hence $\Delta D^{o}_{2,i}/t_{2,i}$ indicates the damage rate.

The quantity $\Delta \bar{D}^{o}_{2,i}$ can easily be determined by model tests. As suggested in [3] the results may, due to lack of more precise formulae, be presented by a stability number S_i and the surf similarity parameter ξ_i [5], defined as follows

$$S_i = \frac{H_{s,i}}{\left(\frac{\overline{W}_i}{\gamma_{B,i}}\right)^{1/3} \left(\frac{\gamma_{B,i}}{\gamma_w} - 1\right)} \tag{25}$$

$$\xi_i = tg\alpha_i \cdot T_{p,i} \left(\frac{g}{2\pi H_{s,i}}\right)^{1/2} \tag{26}$$

where $H_{s,i}$ = significant wave height in front of structure (exclusive reflected waves)
$\overline{W}_i$ = average weight of armour units in basic area i
$\gamma_{B,i}$ = specific weight of armour units in basic area i
γ_w = specific weight of water
α_i = angle of slope at basic area i (α_i < natural angle of repose)
g = acceleration of gravity

Further parameters can easily be included in the formulation if necessary.

For fixed armour unit weight, (25) and (26) represent simply a one-to-one mapping of the basic wave load parameters ($H_{s,i}$, $T_{p,i}$) into a non-dimensional representation, see figure 7.

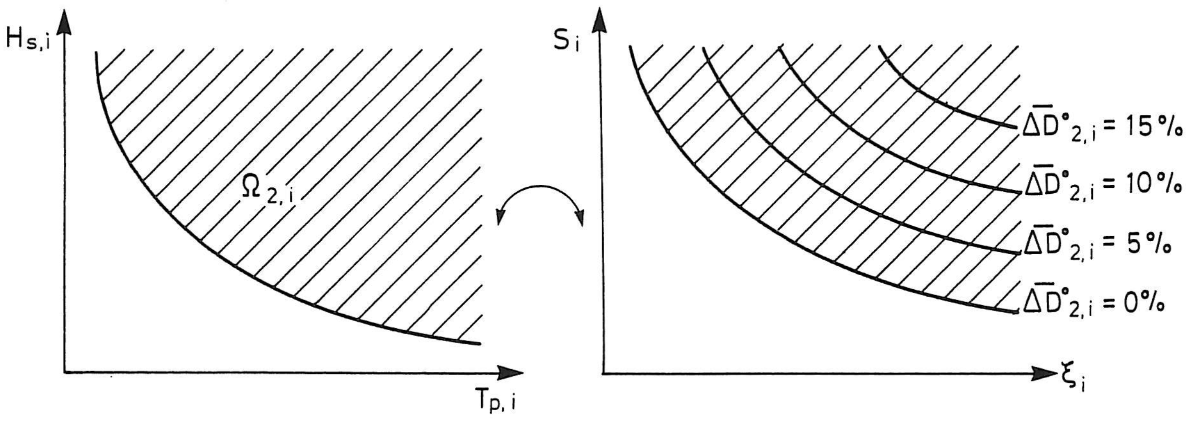


Figure 7: *One-to-one mapping of wave- and stability parameters. a) The domaine of wave parameters giving rise to any damage. b) The stability curves of the equal degree of displacement damage during the interval $t_{2,i}$.*

Stability curves similar to (7b) can also be established for the rocking percentage R_i in the basic unit area i, see figure 10. Note that the rocking percentage is assumed to be independent of the time of exposure, whereas the displacement percentage increases linearly with this quantity.

$T_{0,i}$ and $\Delta D^{\circ}_{2,i}$ in (18) and (24) have been defined as combined stochastic variables, depending on $(H_{s,i}\, , T_{p,i})$. However, a certain scatter is observed in tests when these quantities are determined, because a number of parameters have not been controlled.

The following conditional expectations and conditional variational coefficients are introduced

$$\bar{T}_{0,i}(h) = \left(E\left[\frac{1}{T_{0,i}} \mid (H_{s,i}\, , T_{p,i}) = (h, t(h))\right] \right)^{-1} \quad (27)$$

$$\zeta_i(h) = V\left[\frac{1}{T_{0,i}} \mid (H_{s,i}\, , T_{p,i}) = (h, t(h))\right] \quad (28)$$

$$\Delta \bar{D}^{\circ}_{2,i}(h) = E[\Delta D_{2,i} \mid (H_{s,i}\, , T_{p,i}) = (h, t(h))] \quad (29)$$

$$\kappa_i(h) = V[\Delta D_{2,i} \mid (H_{s,i}\, , T_{p,i}) = (h, t(h))] \quad (30)$$

These quantities are estimated from model tests. The primary contribution to ζ_i is due to the imperfection of the theoretical fatigue model (21). Because of scatter in the limiting wave height $h_{s,i,0}$, determined in hydraulic stability tests, cf. (20), $\bar{\Delta\sigma}_i$ will vary to some extent, even on condition of $H_{s,i}$.

Further the stochastic variables $1/T_{0,i}(h_1)$ and $1/T_{0,i}(h_2)$ respectively $\Delta D_{2,i}(h_1)$ and $\Delta D_{2,i}(h_2)$, are assumed to be fully correlated for every h_1, h_2 corresponding to unchanged relative material response to different sea states.

From (18), (19), (24), (27) - (30) then follows

$$E[\Delta D_{1,i} \mid H_{s,i,max} = \chi] = \frac{t_{0,i}}{\chi - h_1} \int_{h_1}^{\chi} \frac{1}{\bar{T}_{0,i}(u)}\, du \quad (31)$$

$$E[\Delta D_{2,i} \mid H_{s,i,max} = \chi] = \frac{t_{0,i}}{t_{2,i}} \frac{1}{\chi - h_1} \int_{h_1}^{\chi} \Delta \bar{D}^{\circ}_{2,i}(u)\, du \quad (32)$$

$$E[\Delta D^2_{1,i} \mid H_{s,i,max} = \chi] = \left(\frac{t_{0,i}}{\chi - h_1}\right)^2 \int_{h_1}^{\chi} \int_{h_1}^{\chi} \frac{1}{\bar{T}_{0,i}(u_1)} \cdot \frac{1}{\bar{T}_{0,i}(u_2)} (1 + \zeta_i(u_1)\zeta_i(u_2))\, du_1\, du_2 \quad (33)$$

$$E[\Delta D^2_{2,i} \mid H_{s,i,max} = \chi] =$$

$$\left(\frac{t_{0,i}}{t_{2,i}} \frac{1}{\chi - h}\right)^2 \int_{h_1}^{\chi} \int_{h_1}^{\chi} \bar{D}^{\circ}_{2,i}(u_1) \cdot \Delta \bar{D}^{\circ}_{2,i}(u_2)\, (1 + \kappa_i(u_1)\, \kappa_i(u_2))\, du_1\, du_2 \quad (34)$$

The unconditioned expectations $E[\Delta D_{k,i}]$ and $E[\Delta D^2_{k,i}]$ are obtained by taking the expectation of the moments (31) - (34) with respect to the distribution of $H_{s,i,max}$.

The intensities $\lambda_{k,i}$ of the counting processes are obtained from the following expression

$$\lambda_{k,i} = \lambda_0 \int_{\Omega_{k,i}} f_{H_{s,i,max}, T_{p,i,max}}(h,t)\, dh\, dt \tag{35}$$

λ_0 is the expected (average) number of storms per year of engineering significance.

$f_{H_{s,i,max}, T_{p,i,max}}$ is the joint frequency function of the maximum significant wave height $H_{s,i,max}$ and associated peak period $T_{p,i,max}$ during a storm.

The domain of integration $\Omega_{k,i}$ indicates the subset of the sampling space of ($H_{s,i,max}$, $T_{p,i,max}$) for storms, which gives rise to any damage in the k'th failure mode in the basic unit area i. $\Omega_{2,i}$ has been indicated on figure 7, whereas $\Omega_{1,i}$ is given by the following expression, cf. (16), (20), (21)

$$\Omega_{1,i} = \{(h,t) \mid h > h_o\} \tag{36}$$

$$h_0 = \max(h_1, h_{i,min}) \tag{37}$$

$1/\lambda_{k,i}$ is identified as the return period of storms giving rise to damage in the k'th mode in the basic unit area i.

4. THE EXPECTED FAILURE RATE OF BASIC UNIT AREAS

It is assumed that each basic unit area is fully repaired after a failure. Further, the length of the intervals between succeeding failures is assumed to be mutually independent and identically distributed stochastic variables.

The expected number of failures per unit of time $\nu_i : [0,t] \curvearrowright R_+$ in the basic unit area i is then related to the first passage probability density function $f_{L_{1,i}} : [0,t] \curvearrowright R_+$ through the integral equation

$$f_{L_{1,i}}(t) = \nu_i(t) - \int_0^t \nu_i(t-\tau)\, f_{L_{1,i}}(\tau)\, d\tau \tag{38}$$

$\nu_i(t-\tau) f_{L_{1,i}}(\tau)$ represents the joint probability density of a failure at time t, and a first passage failure at time τ, where $0 < \tau < t$. Notice that the failure events have been assumed to be independent. The identity (38) then follows, because the last term on the right hand side is the probability density of failures at time t, which are not first-passages.

By a simple change of integration variable, (38) can be written as follows

$$\nu_i(t) = f_{L_{1,i}}(t) + \int_0^t f_{L_{1,i}}(t-\tau)\, \nu_i(\tau)\, d\tau \tag{39}$$

(39) is an inhomogeneous Volterra integral equation of 2. kind, from which the unknown function $\nu_i(t)$ can be obtained without any numerical difficulties.

From (39) follows immediately

$$\nu_i(0) = f_{L_{1,i}}(0) \tag{40}$$

Further it can be shown that

$$\lim_{t\to\infty} \nu_i(t) = \frac{1}{E[L_{1,i}]} \tag{41}$$

The variation of ν_i at intermediate values has been shown on figure 3.

The expected number of failures per unit of time $\nu_A : [0,t] \curvearrowright R_+$ of the entire armour layer becomes

$$\nu_A(t) = \sum_{i=1}^{M} \nu_i(t) \tag{42}$$

5. THE APPLICATION TO OPTIMUM DESIGN

The repair expenses P_1 during the stipulated lifetime T_0 of the structure, discounted to the time of construction with the inflation-regulated rate of interest r can be written as follows

$$P_1 = \sum_{i=1}^{M} \sum_{\ell=1}^{L_i(T_0)} C_{i,\ell} \cdot (1+r)^{-\tau_{i,\ell}} \tag{43}$$

$\{L_i(t),\ t \in [0, T_0]\}$ are inhomogeneous Poisson counting processes, specifying the random number of failures at times $0 \leqslant \tau_{i,1} < \tau_{i,2} < \ldots\ldots < \tau_{i,L_i(T_0)} \leqslant T_0$ in the basic unit area i. The intensities $\nu_i : [0,T_0] \curvearrowright R_+$ of the counting processes, i.e. the expected number of failures per unit of time, are determined from (39).

$C_{i,\ell}$ is the total inflation-regulated cost of the ℓth failure within basic unit area i, made up of costs of site establishment, down time, social expenses, and repair. These quantities are assumed to be mutual independent stochastic variables, identical distributed as the stochastic variable C_i. C_i depends on the magnitude and duration of the storm at the instant of failure. Consequently C_i may be considered as a combined stochastic variable depending on ($H_{s,i,max}$, $T_{p,i,max}$), i.e.

$$C_i = C_i(H_{s,i,max}, T_{p,i,max}) \tag{44}$$

The characteristic function of the stochastic variable P_1 becomes, cf. (4)

$$M_{P_1}(\theta) = \exp\left(\sum_{i=1}^{M} \int_0^{T_0} \left(M_{C_i}\left((1+r)^{-\tau} \cdot \theta\right) - 1\right) \cdot \nu_i(\tau)\, d\tau\right) \tag{45}$$

$M_{C_i} : R \curvearrowright C$ is the characteristic function of the stochastic variable C_i.

Hence the expectation p_1 and variance of P_1 become

$$p_1 = E[P_1] = \sum_{i=1}^{M} E[C_i] \int_0^{T_0} (1+r)^{-\tau} \cdot \nu_i(\tau)\, d\tau \tag{46}$$

$$\sigma^2_{P_1} = \sum_{i=1}^{M} E[C_i^2] \int_0^{T_0} (1+r)^{-2\tau} \cdot \nu_i(\tau)\, d\tau \tag{47}$$

The moments $E[C_i]$ and $E[C_i^2]$ can be calculated, if the explicit dependency (44) is known.

A shortcoming of (43) is that the rate of interest r will be neither constant nor controlled during the interval $[0,T_0]$. Hence this quantity should rather be modelled as a stochastic process $\{R(t),\ t \in [0,T_0]\}$ with known expectation $\mu_R(0) = r_0$ and zero variance $\sigma_R^2(0) = 0$ at time $t = 0$. Assuming that the rate of interest is continuously ascribed, (43) is replaced by

$$P_1 = \sum_{i=1}^{M} \sum_{\ell=1}^{L_i(T_0)} C_{i,\ell} \cdot \exp\left(-\int_0^{\tau_{i,\ell}} R(u)\, du\right) \tag{48}$$

Because the process $\{R(t)\}$ is independent of the other stochastic variables, (46) is replaced by

$$p_1 = \sum_{i=1}^{M} E[C_i] \int_0^{T_0} E\left[\exp\left(-\int_0^{\tau} R(u)\, du\right)\right] \cdot \nu_i(\tau)\, d\tau \tag{49}$$

For a given design with construction price p_0 the expected repair price p_1 can be calculated from (46) or (49). If p_0 is relatively high, the breakwater will probably have a high reliability, and hence the expected repair costs within a certain period will be low. If, however, p_0 is low, high repair expenses are expected, because the breakwater is correspondingly weak. Consequently p_0 will be a monotonic drecreasing function of p_1. It then follows that the expected total expenses of the breakwater, made up of construction price p_0 and maintenance expenses p_1, have a minimum, when considered as a function of p_1, cf. figure 8.

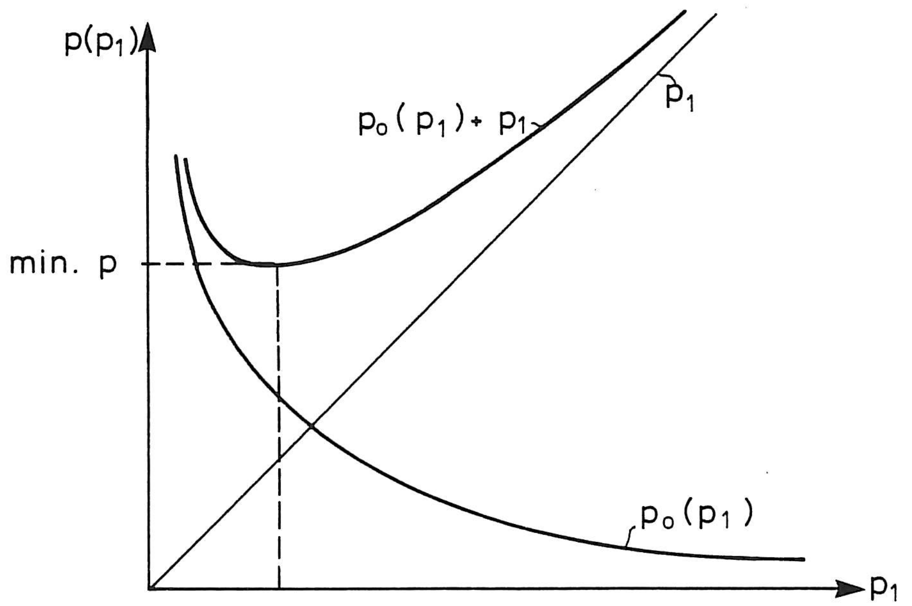


Figure 8: The variation of construction price p_0 plus the expected maintenance expenses p_1 as a function of p_1.

6. NUMERICAL EXAMPLE

The outlined theory will be demonstrated for a breakwater specified by the succeeding parameters

Slope of angle	:	$\tan\alpha = 1/1.5$
Type of armour	:	Complex, slender unreinforced concrete units $\gamma_B = 23.8$ kN/m³ Dynamic tensile strength $\bar{\sigma}_T = 6000$ kPa
Reference armour unit	:	$\bar{W}_0 = 150$ kN Number of units = 0.17/m²
Unit area	:	32 m x 32 m = 1024 m²
Number of unit areas	:	M = 20
Length of breakwater	:	$\ell = 20 \times 32 = 640$ m
Stipulated lifetime	:	$T_0 = 100$ years
Specific weight of water	:	$\gamma_w = 10.0$ kN/m³
Long term distribution of maximum wave heights, eq. (16)	:	$h_1 = 0.1$ m $h_2 = 6.0$ m $h_3 = 2.8$
Duration of storms, eq. (19)	:	$t_{0,i} = 70$ hours
Stress range relation, eq. (20)	:	$h_{s,i,1} = 12.9$ m $\Delta\bar{\sigma}_{i,1} = 6000$ kPa
Fatigue parameters, eq. (21)	:	$m_i = 6.36$ $\Delta\bar{\sigma}_{i,min} = 1250$ kPa

The corresponding Wöhler curve, obtained from test series with 5 repeated tests is shown on figure 9.

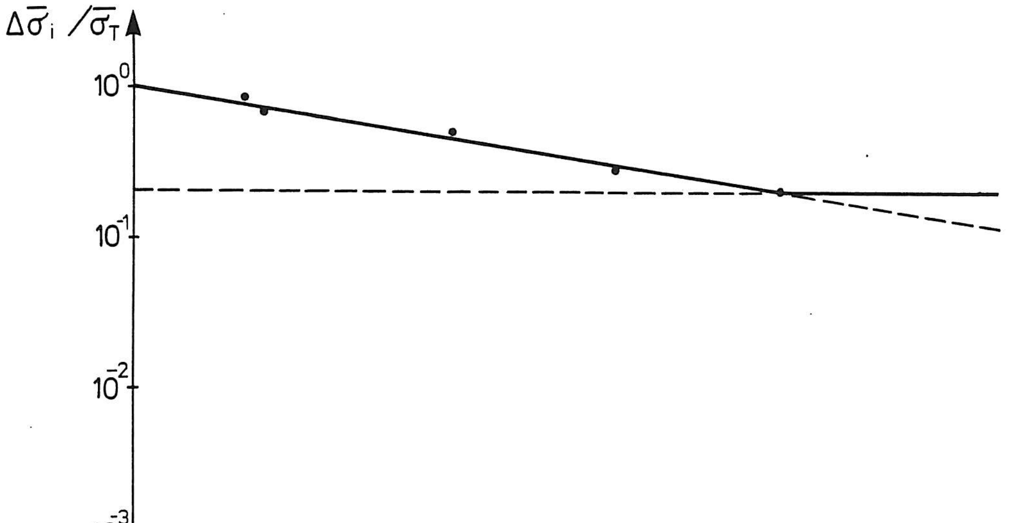


Figure 9: *Wöhler curve for impact loaded armour units (flexural stress). Number of stress ranges to first sign of crack. Average value from 5 tests (Burcharth 1983).*

The variational coefficient ζ_i of $T_{0,i}/T_{z,i}$, estimated from the above test-series, has been indicated in table 1 as a function of $\Delta\bar{\sigma}/\Delta\bar{\sigma}_T$.

Table 1: *The variational coefficient of fatigue life duration as a function of average stress range.*

$\Delta\bar{\sigma}$ / $\bar{\sigma}_T$	$\zeta_i\,(\Delta\bar{\sigma})$
0.208	2.103
0.302	1.550
0.500	1.171
0.698	0.146
0.854	0.404

Despite the scatter there is a tendency of increased variational coefficients as $\Delta\bar{\sigma}$ decreases. The functional relationship has been represented by the following linear fit:

$$\zeta_i = 2.3 - 2.1 \frac{\Delta\bar{\sigma}_i}{\bar{\sigma}_T}$$

The uncertainty contributions from the determination of the limiting wave height $h_{s,i,0}$ have been ignored in the present study.

Peak period relation, eq. (22)	:	$t_{p,i,1} = 16.7$ s
		$h_4 = 0.45$
Zero crossing period relation, eq. (23)	:	$t_{z,i,1} = 11.5$ s
Intensity of significant storms	:	$\lambda_0 = 0.85$/year

The average rocking percentage $\bar{R}$ and the average damage increment $\Delta\bar{D}^{\circ}_{2,i}$ and the associated variational coefficient κ_i have been determined by model tests, see figure 10. Notice that the damage was not equally distributed over the quadratic area. The damage within a zone of 25% amounted to approximately the double of the indicated values.

The slope of the stability curves has been taken from [3]. The stability curve corresponding to p percentage of damage can be formulated analytically as follows

$$S_i = s_{p,i} - r_{0,i}\,\xi_i \quad , \quad r_{0,i} = 0.325 \tag{50}$$

where $s_{p,i}$ has been tabulated below.

From a given wave height $H_{s,i}$ and armour unit weight $\bar{W}$, S_i and ξ_i are calculated from (25) and (26). $s_{p,i}$ is then obtained from (50), and the rocking percentage $\bar{R}$ = p, respectively damage percentage $\Delta\bar{D}^{\circ}_{2,i}$ = p and variational coefficient κ_i, are obtained by linear interpolation or extrapolation from table 2.

Legend: Each data set obtained from 20 tests

	S_i	ξ_i	Rocking R_i in % μ	Rocking R_i in % σ	Displacement $\Delta D^{\circ}_{2,i}$ in % μ	Displacement $\Delta D^{\circ}_{2,i}$ in % σ
△	1.67	4.30	1.46	0.90	0.11	0.32
▲	2.26	3.68	3.28	1.74	0.54	0.79
○	2.58	3.46	5.03	2.57	1.73	2.26
●	3.19	3.10	7.21	2.82	3.45	2.74
×	3.47	2.98	10.15	3.16	5.98	3.24

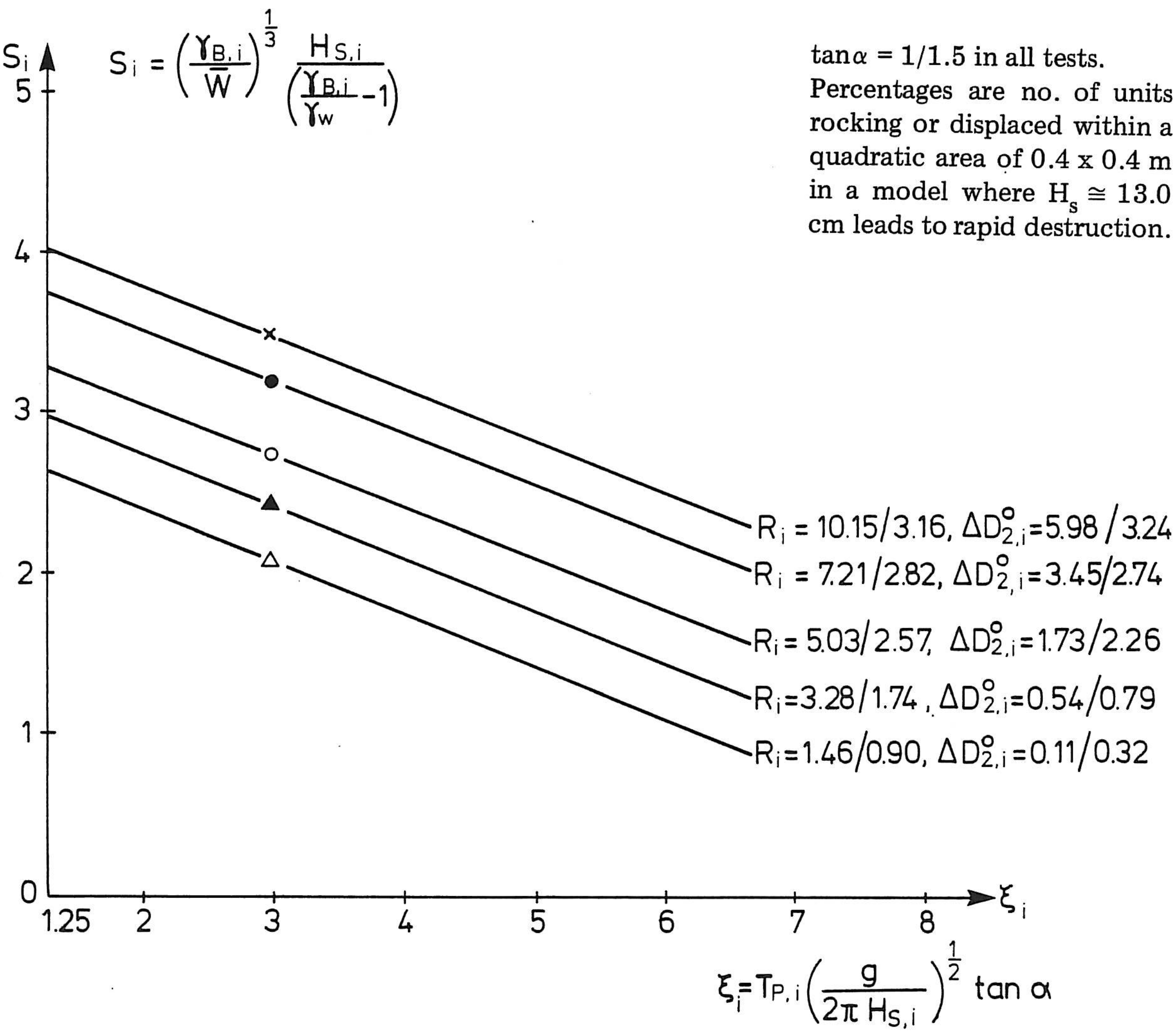


Figure 10: Hydraulic rocking and displacement stability curves. Damage after $t_{2,i} \cong 3$ hours in prototype, corresponding to 1200 waves with expected zero crossing period $T_{z,i} \sim 9$ s. (Burcharth and Brejnegaard 1982, 1983).

Table 2: Hydraulic stability data.

	Rocking		Displacement	
$s_{p,i}$	p (%)	κ_i	p (%)	κ_i
3.07	1.46	0.616	0.11	2.91
3.46	3.28	0.530	0.54	1.46
3.70	5.03	0.511	1.73	1.306
4.20	7.21	0.391	3.45	0.794
4.48	10.15	0.311	5.98	0.542

$\eta^{\circ}_{1,i} = 0.1$. Hence the 10% rocking stability curve is determined by $s_{p,i} = 4.466$.

For $\bar{W}_i = \bar{W}_0 = 150$ kN follows from (22), (25), (26), (50): $h_{s,i,0} = 8.100$ m.

From (20), (21) then follows : $h_{i,min} = 9.100$ m.

From (37) : $h_0 = \max(0.1, 9.100) = 9.100$ m.

The 0% displacement stability curve is determined by $s_{p,i} = 2.970$.

For $\bar{W} = \bar{W}_0 = 150$ kN follows from (22), (25), (26), (50) : h = 4.177 m.

From (35) follows

$$\lambda_{1,i} = 0.85 \exp\left(-\left(\frac{9.100 - 0.1}{6.0}\right)^{2.8}\right) = 0.0378 \text{ / year}$$

$$\lambda_{2,i} = 0.85 \exp\left(-\left(\frac{4.176 - 0.1}{6.0}\right)^{2.8}\right) = 0.606 \text{ / year}$$

The moments of the damage rates become

$$E[\Delta D_{1,i}] = 1.618$$

$$E[\Delta D_{2,i}] = 0.07785$$

$$E[\Delta D^2_{1,i}] = 4345$$

$$E[\Delta D^2_{2,i}] = 0.04039$$

When $\bar{W}_i$ is varied, these quantities will vary proportionally.

The allowable damage limits $\eta_{k,i}$, $\eta^{\circ}_{1,i}$, and the functions $s_{k,i}$ in (1) and (21) have been selected as follows

$$\eta_{1,i} = 1 \quad , \quad \eta^{\circ}_{1,i} = 0.1$$

$$\eta_{2,i} = 0.1$$

$$s_{k,i} \equiv 1 \quad , \quad k \in 1, 2$$

The first passage density function of a basic unit area can now be calculated. The result has been shown on figure 11 as a function of the armour unit weight. Notice the shift of the probability mass towards higher failure times, indicating increased reliability, as the average armour unit weight is increased.

As an example the construction price of the entire breakwater is taken as dependent on the average armour unit weight as follows

$$p_0 = (a + b\left(\frac{\bar{W}}{\bar{W}_0}\right)^{\frac{1}{3}})\ell \quad , \quad a = 0.35 \cdot 10^6 \text{ Dkr./m}, \quad b = 0.15 \cdot 10^6 \text{ Dkr./m}$$

The expected inflation regulated repair price of a basic unit area with length 32 m has been assumed to be

$$E[C_i] = 32\text{m} \cdot c\left(\frac{\bar{W}}{\bar{W}_0}\right)^{\frac{1}{3}} \quad , \quad c = 0.2 \cdot 10^6 \text{ Dkr./m}$$

The authors are fully aware of the simplicity of these cost estimates. Down-time costs should of course be included in a real case.

The expected total expenses made up of construction price and expected repair prices can now be calculated for a specific rate of interest r and an average armour unit weight $\bar{W}$. These relationship have been clarified on figure 12. The minimum of the curves specifies the optimum armour unit weight W_{opt}, according to the applied design criteria.

On figure 13 is shown the dependency of optimum design W_{opt} on the rate of interest r. When the rate of interest r is low, the repair expenses should be minimized and hence W_{opt} is high.

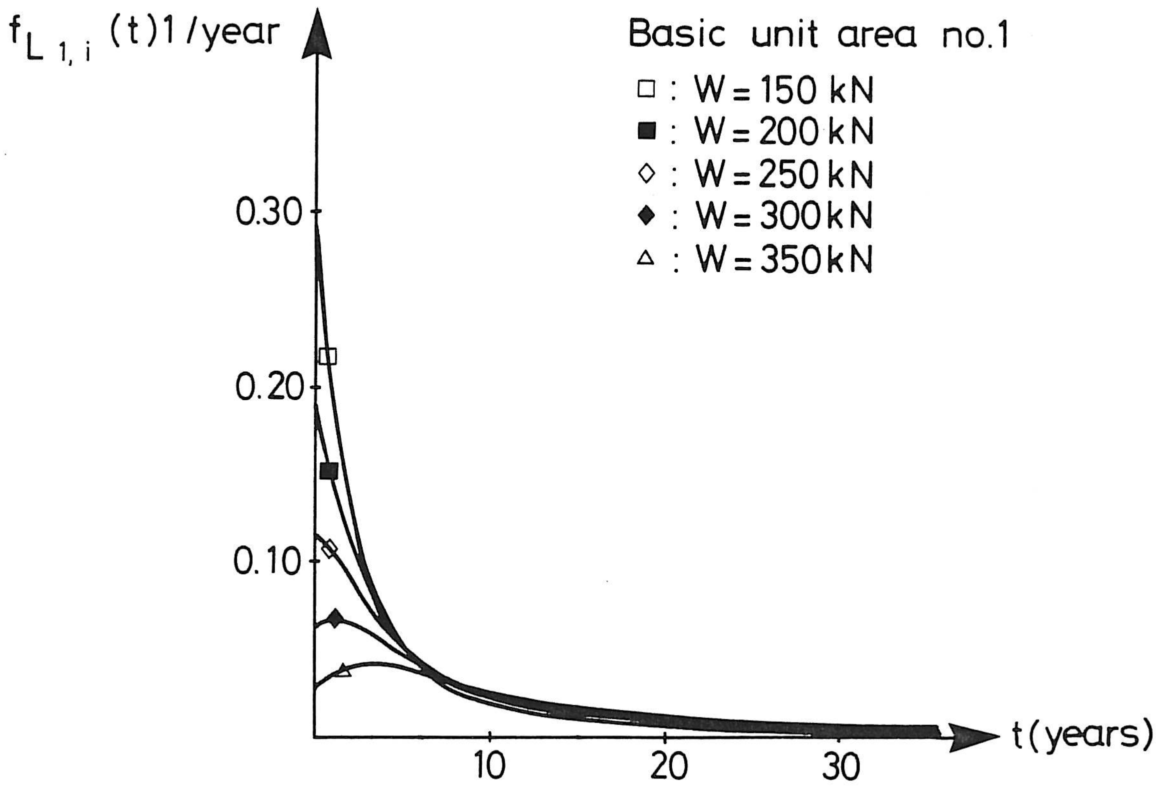


Figure 11: First passage densities as a function of the average weight of armour units in the basic unit area i = 1.

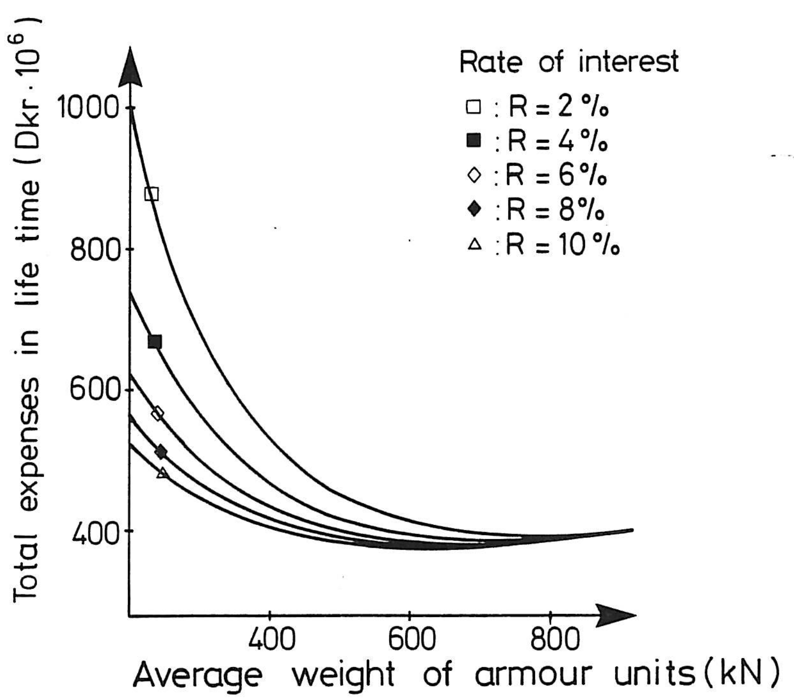


Figure 12: *The variation of expected total expenses of breakwater armour layer during stipulated life time T_0 = 100 year, depending on armour unit weight $\bar{W}$ and inflation regulated rate of interest r.*

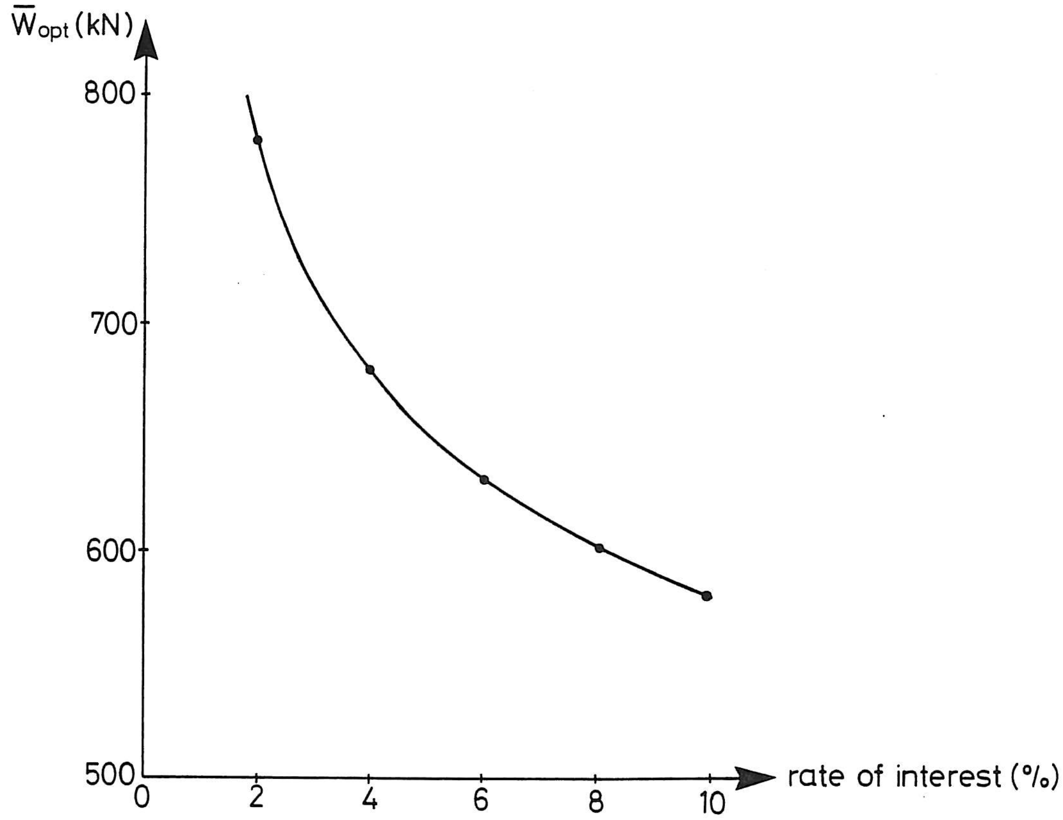


Figure 13: *Optimal armour unit weight as a function of the inflation regulated rate of interest during the stipulated lifetime.*

7. CONCLUDING REMARKS

A level III reliability method has been developed from which the armour layer of a rubble mound breakwater can be designed, so that the total costs made up of construction price and expected maintenance expenses are minimized. The theory was demonstrated by a numerical example, where the armour unit weight is the only design parameter. The cost estimates for construction and repair used in the numerical example are primitive and used as an illustrative example only. More evaluated cost functions should be implemented in actual applications. An extension to more complex problems where several parameters (slope of angle, material strength parameters etc., etc.) are introduced in the cost function, is straight forward. A computer programme has been developed, from which all relevant quantities in the theory can be determined.

8. REFERENCES

[1] GRAVESEN, H., and SØRENSEN, T.: »Stability of Rubble Mound Breakwaters«, Proc. 24th Int. Nav. Congress, PIANC, Leningrad, 1977, p. 65-74.

[2] GRAVESEN, H., JENSEN, O. J., and SØRENSEN, T.: »Stability of Rubble Mound Breakwaters II«, Danish Hydraulic Institute (DHI), 1980.

[3] BURCHARTH, H. F.: »The Effect of Wave Grouping on On-Shore Structures«, Coastal Engineering, 2 (1979), p. 189-199.

[4] PARZEN, E.: »Stochastic Processes«, Holden-Day, San Fransisco, 1962.

[5] BRUUN, P., and GÜNBAK, A. R.: »New Design Principles for Rubble Mound Structures«, Proc. Int. Coastal Eng. Conf., 15th, 1976.